A Lower Bound on Multi-hop Transmission Delay in Cognitive Radio Ad Hoc Networks

Weng Chon Ao* and Shin-Ming Cheng†
*Department of Electrical Engineering, University of Southern California. Email: wao@usc.edu
†Department of Computer Science and Information Engineering,
National Taiwan University of Science and Technology, Taipei 106, Taiwan. Email: smcheng@mail.ntust.edu.tw

***Abstract*—This paper analyzes the delay of multi-hop transmissions in cognitive radio ad hoc networks (CRNs) to investigate the performance of routings in CRN. Compared to routing in ad hoc networks, additional medium access delay is introduced in CRN since opportunistic transmissions of secondary users (SUs) should not violate the interference constraints at primary receivers. Moreover, additional retransmission delay occurs in CRN because received signals at SUs are interfered with by both primary transmitters and concurrent secondary transmitters. We propose an analytically tractable model to investigate routings in CRN considering medium access delay, retransmission delay and the hop count of the end-to-end route. Through optimizing the number of hops of the end-to-end route, a lower bound on the end-to-end packet transmission delay is developed, which facilitates delay QoS provisioning and rate-delay trade-off in CRN. Consequently, this research serves as the valid framework for baseline performance analysis and offers novel avenues to routing design in CRN.**

I. Introduction

Cognitive radio (CR) is a promising technology to improve the utilization of scarce radio spectrum by sensing and opportunistically accessing the spectrum of primary systems [1]. Two main spectrum sharing approaches (i.e., interweave and underlay) are proposed [2]. In the interweave paradigm, secondary users (SUs) orthogonally access the spectrum holes of primary systems in time and frequency facilitated by advanced spectrum sensing techniques. In the underlay paradigm, primary users (PUs) and SUs transmit at the same time and in the same frequency band, while the interference generated by SUs to PUs is constrained.

To concatenate CRs as a cognitive radio ad hoc network (CRN), Quality of Service (QoS) provisioning for end-to-end packet transmissions is an essential must. However, the following delays challenge routing toward QoS guarantee in underlay CRN, which consists of medium access delay, retransmission delay, and the hop count of the end-to-end route.

- Medium access delay: The cognitive principle states that SUs are aware of avoiding unacceptable interference to PUs. Given outage constraints of primary receivers (PRs), the density of active secondary transmitters (STs) should be limited to maintain the outage constraints of PRs, leading to a decrease in the access probability of an SU. Additional medium access delay is thus introduced compared to that in traditional ad hoc networks.
- Retransmission delay: Since SUs are transparent to PUs in CRN, the received signal of a secondary receiver (SR) further suffers inter-system interference generated by primary transmitters (PTs). The degradation of the received signal quality at an SR would cause transmission outage more often and thus increase the number of retransmissions.
- Hop count of the end-to-end route: Since a packet may be transmitted from the source node to the destination node over multiple hops, the delay increases with the number of hops of the end-to-end route.

Note that the above factors may influence each other. For example, when we use long-hop transmissions (i.e., the hop count becomes smaller), the link reliability decreases due to the increasing hop-length, thus causing more retransmissions. Also, as the active probability of an SU increases (i.e., the medium access delay becomes smaller), the intra-system interference increases, causing more retransmissions. These observations reveal trade-offs among different types of delay.

In this paper, we analytically derive the medium access delay and the retransmission delay of packet transmission in the underlay CRN, considering both intra-system interference among SUs and inter-system interference from/to PUs. A lower bound on the end-to-end packet transmission delay is obtained by optimizing the hop-length (or the hop count) of the end-to-end route. Optimal transmit power allocation is further investigated.

The remainder of this paper is organized as follows. Section II summarizes the current routings in interweave and underlay CRNs. In Section III, the network model is introduced, and the interference at PR/SR is characterized to obtain the outage probability at PR/SR. In Section IV, the lower bound on the average end-to-end packet transmission delay is derived. Section V gives the conclusion.

II. Related Works

Some related works on routing in interweave and underlay CRNs are summarized as follows.

- Interweave: Research of routing in the interweave CRN focuses on the modeling of spectrum availability. Spectrum availability depends on the PU activity and should be considered as the performance metric in the route computation [3], [4]. A probabilistic metric is proposed in [5], [6] to capture the PU behavior. In [7], a path is determined based on the prediction of available durations of secondary links. In [8], a path with the highest connectivity, which is evaluated by using statistics of PU activity, is chosen for routing.

- Underlay: In the underlay CRN (a kind of heterogeneous ad hoc network with intra-system and inter-system interference and priority), we should focus on interference modeling [9]–[11]. Research of routing in homogeneous ad hoc networks considering intra-system interference is developed in [12], [13]. Bounds on the speed of information propagation are derived in [14], [15]. Furthermore, the effects of ARQ, queueing delay, and multiple routes on routing performance are respectively addressed in [16], [17], and [18]. However, the delay analysis of routing in underlay CRN has not received much investigation.

III. Network model

A. Secondary ad hoc network

We consider a secondary ad hoc network (denoted as SN), in which the spatial distribution of SUs is assumed to follow a homogeneous Poisson Point Process (PPP) with density λ. Let $\Phi = \{X_k\}$ denote the set of locations of the SUs and let P_{SN} denote the transmit power of an SU. The medium access control protocol of the secondary network is assumed to be slotted ALOHA with medium access probability p, i.e., each SU is allowed to transmit (be active) with probability p. The set of active SUs is a PPP with density $p\lambda$ and is denoted as $\Phi^1 = \{X_k \in \Phi : B_k(p) = 1\}$, where $B_k(p)$ are independent and identically distributed (i.i.d.) Bernoulli random variables with parameter p associated with X_k. The information rate of a secondary transmission is denoted as R_{SN}.

B. Primary ad hoc network

The secondary network coexists with a primary ad hoc network (denoted as PN), where the spatial distribution of PTs is assumed to follow a PPP with density μ. Each PT has a dedicated PR at distance d away with an arbitrary direction so that the PRs also form a PPP with density μ. The set of locations of the PTs is denoted as $\Psi = \{Y_k\}$ and the transmit power of a PT is P_{PN}. The information rate of a primary transmission is denoted as R_{PN}. Each PR has an outage constraint with maximum outage probability ϵ.

C. Inter-system and Intra-system interference

The inter-system interference from the PTs to a typical SU (a reference SU located at the origin) is denoted as $I_{PN,SN} = \sum_{p\in\Psi} G_{p,s} P_{PN} \|p\|^{-\alpha}$. $G_{p,s}$ denotes the fading gain of the interfering link from the PT at p to the typical SU, which is assumed to be independently drawn from an exponential distribution (i.e., Rayleigh fading) with mean $m_{p,s}$. $\|p\|$ is the distance between the PT at p and the typical SU, and α is the path loss exponent. Furthermore, the intra-system interference from the PTs to a typical PR is denoted as $I_{PN,PN} = \sum_{p\in\Psi} G_{p,p} P_{PN} \|p\|^{-\alpha}$, where $G_{p,p}$ denotes the fading gain of the interfering link from the PT at p to the typical PR and is independently drawn from an exponential distribution with mean $m_{p,p}$.

Similarly, the inter-system interference from the active SUs to a typical PR is denoted as $I_{SN,PN} = \sum_{s\in\Phi^1} G_{s,p} P_{SN} \|s\|^{-\alpha}$, where $G_{s,p}$ denotes the fading gain of the interfering link from the SU at s to the typical PR and is independently drawn from an exponential distribution with mean $m_{s,p}$. Furthermore, the intra-system interference from the active SUs to a typical SU is denoted as $I_{SN,SN} = \sum_{s\in\Phi^1} G_{s,s} P_{SN} \|s\|^{-\alpha}$, where $G_{s,s}$ denotes the fading gain of the interfering link from the SU at s to the typical SU and is independently drawn from an exponential distribution with mean $m_{s,s}$. All packet transmissions of the networks are assumed to be slotted and synchronized.

D. Outage probability

The outage probability at PRs and SRs will be used to compute the medium access delay and the retransmission delay later. Please note that the derivation of outage probability for the general fading channel model is provided in the Appendix.

1) Outage probability at PR: For a typical PR, outage occurs when the channel between the PR and its dedicated PT cannot support the information rate R_{PN}, i.e.,

$$\log\left(1 + \frac{G_{p,p} P_{PN} d^{-\alpha}}{N_0 + I_{PN,PN} + I_{SN,PN}}\right) < R_{PN}, \qquad (1)$$

where $G_{p,p}$ is the fading gain of the desired link between the PT and the PR and is drawn from an exponential distribution with mean $m_{p,p}$, d is the distance between the PT-PR pair, and N_0 is the background noise power.

The Laplace transforms of the interference $I_{PN,PN}$ and $I_{SN,PN}$ (2D Poisson shot noise processes [19]) are respectively

$$\begin{aligned} &\mathbb{E}[e^{-sI_{PN,PN}}] = \exp(-\mu\pi P_{PN}^{\delta} \mathbb{E}[G_{p,p}^{\delta}]\Gamma(1-\delta)s^{\delta}), \\ &\mathbb{E}[e^{-sI_{SN,PN}}] = \exp\left(-p\lambda\pi P_{SN}^{\delta} \mathbb{E}[G_{s,p}^{\delta}]\Gamma(1-\delta)s^{\delta}\right), \end{aligned} \qquad (2)$$

where $\delta = \frac{2}{\alpha}$, $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$ is the Gamma function, $\mathbb{E}[G_{p,p}^{\delta}] = m_{p,p}^{\delta}\Gamma(1+\delta)$, and $\mathbb{E}[G_{s,p}^{\delta}] = m_{s,p}^{\delta}\Gamma(1+\delta)$. As a result, the outage probability at the typical PR can be computed as

$$\begin{aligned} &\mathbb{P}\left(\log\left(1 + \frac{G_{p,p} P_{PN} d^{-\alpha}}{N_0 + I_{PN,PN} + I_{SN,PN}}\right) < R_{PN}\right) \\ &= 1 - \mathbb{P}(G_{p,p} \geq P_{PN}^{-1} d^{\alpha} (2^{R_{PN}} - 1)(N_0 + I_{PN,PN} + I_{SN,PN})) \\ &\overset{(a)}{=} 1 - e^{-m_{p,p}^{-1} P_{PN}^{-1} d^{\alpha}(2^{R_{PN}}-1)N_0} \mathbb{E}[e^{-m_{p,p}^{-1} P_{PN}^{-1} d^{\alpha}(2^{R_{PN}}-1)I_{PN,PN}}] \\ &\quad \cdot \mathbb{E}[e^{-m_{p,p}^{-1} P_{PN}^{-1} d^{\alpha}(2^{R_{PN}}-1)I_{SN,PN}}] \\ &\overset{(b)}{=} 1 - \exp\left\{-m_{p,p}^{-1} P_{PN}^{-1} d^{\alpha}(2^{R_{PN}}-1)N_0\right\} \\ &\quad \cdot \exp\left\{-\left(\mu P_{PN}^{\delta}\mathbb{E}[G_{p,p}^{\delta}] + p\lambda P_{SN}^{\delta}\mathbb{E}[G_{s,p}^{\delta}]\right)\pi\Gamma(1-\delta)\right. \\ &\quad \left. \cdot \cdot m_{p,p}^{-\delta} P_{PN}^{-\delta} d^2 (2^{R_{PN}}-1)^{\delta}\right\}, \end{aligned} \qquad (3)$$

where (a) follows by $\mathbb{P}(G_{p,p} \geq x) = e^{-m_{p,p}^{-1}x}$, and (b) follows by (2).

2) Outage probability at SR: For a typical SR, outage occurs when the channel between the SR and an ST cannot support the information rate R_{SN}, i.e.,

$$\log\left(1+\frac{G_{s,s}P_{SN}r^{-\alpha}}{N_0+I_{PN,SN}+I_{SN,SN}}\right)<R_{SN}, \qquad (4)$$

where $G_{s,s}$ is the fading gain of the desired link between the ST and the SR and is drawn from an exponential distribution with mean $m_{s,s}$, r is the distance between the ST and the SR.

The Laplace transforms of the interference $I_{PN,SN}$ and $I_{SN,SN}$ are respectively

$$\begin{aligned}\mathbb{E}[e^{-sI_{PN,SN}}]&=\exp\left(-\mu\pi P_{PN}^{\delta}\mathbb{E}[G_{p,s}^{\delta}]\Gamma(1-\delta)s^{\delta}\right),\\ \mathbb{E}[e^{-sI_{SN,SN}}]&=\exp\left(-p\lambda\pi P_{SN}^{\delta}\mathbb{E}[G_{s,s}^{\delta}]\Gamma(1-\delta)s^{\delta}\right),\end{aligned} \qquad (5)$$

where $\mathbb{E}[G_{p,s}^{\delta}]=m_{p,s}^{\delta}\Gamma(1+\delta)$, and $\mathbb{E}[G_{s,s}^{\delta}]=m_{s,s}^{\delta}\Gamma(1+\delta)$. Consequently, the outage probability at the SR can be computed as

$$\begin{aligned}&\mathbb{P}\left(\log\left(1+\frac{G_{s,s}P_{SN}r^{-\alpha}}{N_0+I_{PN,SN}+I_{SN,SN}}\right)<R_{SN}\right)\\ &=1-\exp\left\{-m_{s,s}^{-1}P_{SN}^{-1}r^{\alpha}(2^{R_{SN}}-1)N_0\right\}\\ &\quad\cdot\exp\left\{-\left(\mu P_{PN}^{\delta}\mathbb{E}[G_{p,s}^{\delta}]+p\lambda P_{SN}^{\delta}\mathbb{E}[G_{s,s}^{\delta}]\right)\right.\\ &\quad\left.\cdot\,\pi\Gamma(1-\delta)m_{s,s}^{-\delta}P_{SN}^{-\delta}r^{2}(2^{R_{SN}}-1)^{\delta}\right\}.\end{aligned} \qquad (6)$$

E. Maximum medium access probability of an SU

To avoid interference from the active SUs violating the outage constraints ϵ at PRs, the maximum medium access probability of an SU (denoted as $\tilde{p}$) should satisfy

$$\begin{aligned}&\mathbb{P}\left(\log\left(1+\frac{G_{p,p}P_{PN}d^{-\alpha}}{N_0+I_{PN,PN}+I_{SN,PN}}\right)<R_{PN}\right)\\ &=1-\exp\left\{-m_{p,p}^{-1}P_{PN}^{-1}d^{\alpha}(2^{R_{PN}}-1)N_0\right\}\\ &\quad\cdot\exp\left\{-\left(\mu P_{PN}^{\delta}\mathbb{E}[G_{p,p}^{\delta}]+\tilde{p}\lambda P_{SN}^{\delta}\mathbb{E}[G_{s,p}^{\delta}]\right)\pi\Gamma(1-\delta)\right.\\ &\quad\left.\cdot\,m_{p,p}^{-\delta}P_{PN}^{-\delta}d^{2}(2^{R_{PN}}-1)^{\delta}\right\}=\epsilon.\end{aligned} \qquad (7)$$

As a result, $\tilde{p}$ can be computed as

$$\begin{aligned}\tilde{p}=&\left(\frac{-\ln(1-\epsilon)-m_{p,p}^{-1}P_{PN}^{-1}d^{\alpha}(2^{R_{PN}}-1)N_0}{\pi\Gamma(1-\delta)m_{p,p}^{-\delta}P_{PN}^{-\delta}d^{2}(2^{R_{PN}}-1)^{\delta}}\right.\\ &\left.-\,\mu P_{PN}^{\delta}\mathbb{E}[G_{p,p}^{\delta}]\right)\left(\lambda P_{SN}^{\delta}\mathbb{E}[G_{s,p}^{\delta}]\right)^{-1}.\end{aligned} \qquad (8)$$

The above equation holds when $\mu<\frac{-\ln(1-\epsilon)-m_{p,p}^{-1}P_{PN}^{-1}d^{\alpha}(2^{R_{PN}}-1)N_0}{\pi\Gamma(1-\delta)m_{p,p}^{-\delta}d^{2}(2^{R_{PN}}-1)^{\delta}\mathbb{E}[G_{p,p}^{\delta}]}$, indicating that the intra-system interference from PTs is not strong enough to cause violation of the outage constraints at PRs.

IV. Lower Bound on the Delay of Routing in Secondary Network

The end-to-end packet transmission delay of routing in the secondary ad hoc network consists of medium access delay, retransmission delay, and the hop count of the end-to-end route. A lower bound on the end-to-end delay is obtained by optimizing the hop-length (or the hop count) of the end-to-end route, where we assume that hops are separated with equal distance on the line between a source node and a destination node. Let L denote the distance between the source node and the destination node, and let M denote the number of hops of the end-to-end route. The relay nodes are thus separated by the same distance L/M.

A. Medium access delay

The probability that the transmitter node of a hop becomes active is $p,\ 0\le p\le\tilde{p}$, where $\tilde{p}$ is computed in (8). That is, given outage constraints ϵ at PRs, the maximum medium access probability of an SU is limited to $\tilde{p}$. Here, we re-parameterize the access probability of an SU as $q\tilde{p},\ 0\le q\le 1$. Thus, the average medium access delay of an SU is $1/(q\tilde{p})$.

B. Retransmission delay

Retransmission occurs when the receiver node of a hop is outage, as shown in Section III-D2. The success (non-outage) probability of a transmission (denoted as p^s) can be computed by using (6) with $r=L/M$ and $p=q\tilde{p}$, i.e.,

$$\begin{aligned}p^{s}=&\exp\left\{-m_{s,s}^{-1}P_{SN}^{-1}(L/M)^{\alpha}(2^{R_{SN}}-1)N_0\right\}\\ &\cdot\exp\left\{-\left(\mu P_{PN}^{\delta}\mathbb{E}[G_{p,s}^{\delta}]+q\tilde{p}\lambda P_{SN}^{\delta}\mathbb{E}[G_{s,s}^{\delta}]\right)\pi\Gamma(1-\delta)\right.\\ &\left.\cdot\,m_{s,s}^{-\delta}P_{SN}^{-\delta}(L/M)^{2}(2^{R_{SN}}-1)^{\delta}\right\}.\end{aligned} \qquad (9)$$

Channel outages are supposed to be independent across hops and across retransmissions. In other words, the spatial and temporal correlation of interference observed at different nodes are ignored due to the fact that the fading gains of the desired and interfering signals are i.i.d. and they pick up different realizations at each slot [20]. As a result, the average number of retransmissions is $1/p^s$.

C. End-to-end packet transmission delay

The number of trials (time slots) of a relay node to access the medium and transmit a packet successfully to the next hop is a geometric random variable (denoted as $T_i,\ 1\le i\le M$) with parameter $q\tilde{p}p^s$. The mean of T_i is $1/(q\tilde{p}p^s)$. The end-to-end packet transmission delay (denoted as T) is $T=\sum_{i=1}^{M}T_i$, which is the sum of M i.i.d. geometric random variables. T follows the negative binomial distribution with parameters M and $q\tilde{p}p^s$, and its ccdf (complementary cumulative distribution function) is

$$\begin{aligned}\mathbb{P}(T>t)&=1-\sum_{j=M}^{t}\binom{j-1}{M-1}(1-q\tilde{p}p^s)^{j-M}(q\tilde{p}p^s)^{M}\\ &=1-I_{q\tilde{p}p^s}(M,t-M+1),\end{aligned} \qquad (10)$$

where $I_x(a,b)$ is the regularized incomplete beta function.

Consider a delay QoS constraint $\mathbb{P}(T>\tau)\le\sigma$, which states that the probability that the end-to-end delay exceeds the threshold τ should be less than or equal to σ. The maximum information rate satisfying the QoS constraint $R_{SN}(\sigma)$ can be obtained by solving $\mathbb{P}(T>\tau)=\sigma$. Intuitively, when the information rate increases, the probability of channel outage (and the number of retransmissions) increases and thus the end-to-end delay increases. There exists a trade-off between the end-to-end delay and the information rate. A tight delay QoS constraint with small σ leads to a small $R_{SN}(\sigma)$. Using (10), the maximum information rate $R_{SN}(\sigma)$ can be derived by solving $I_{q\tilde{p}p^s}(M,\tau-M+1)=1-\sigma$.

Furthermore, we note that by Markov's inequality, we have $\mathbb{P}(T > \tau) \leq \tau^{-1}\mathbb{E}[T]$, where $\mathbb{E}[T] = M/(q\tilde{p}p^s)$ is the average end-to-end packet transmission delay. $\mathbb{E}[T] \leq \tau\sigma$ serves as the sufficient condition for the delay QoS constraint. In this sense, minimizing the average end-to-end delay with respect to the hop count, the transmit power, and the medium access probability, as discussed in the following, benefits the rate-delay tradeoff.

1) Optimal hop count: The average end-to-end packet transmission delay $\mathbb{E}[T]$ is minimized with respect to the number of hops M. When the background noise power N_0 is ignored (i.e., $N_0 = 0$), the optimization problem can be simplified and formulated as

$$\begin{aligned} \text{minimize} \quad & \mathbb{E}[T] = \frac{M}{q\tilde{p}p^s} = Mq^{-1}k_0^{-1}P_{SN}^{\delta} \\ & \exp\left\{(k_1 + qk_2)P_{SN}^{-\delta}M^{-2}\right\} \\ \text{subject to} \quad & M \geq 1, \end{aligned} \tag{11}$$

where $\tilde{p} = k_0 P_{SN}^{-\delta}$, $p^s = \exp\left\{-(k_1 + qk_2)P_{SN}^{-\delta}M^{-2}\right\}$, $k_0 \triangleq \left(\frac{-\ln(1-\epsilon)}{\pi\Gamma(1-\delta)m_{p,p}^{-\delta}d^2(2^{R_{PN}}-1)^{\delta}} - \mu\mathbb{E}[G_{p,p}^{\delta}]\right)\left(\lambda\mathbb{E}[G_{s,p}^{\delta}]\right)^{-1}$ P_{PN}^{δ}, $k_1 \triangleq \mu P_{PN}^{\delta}\mathbb{E}[G_{p,s}^{\delta}]\pi\Gamma(1-\delta)m_{s,s}^{-\delta}L^2(2^{R_{SN}}-1)^{\delta}$, and $k_2 \triangleq k_0\lambda\mathbb{E}[G_{s,s}^{\delta}]\pi\Gamma(1-\delta)m_{s,s}^{-\delta}L^2(2^{R_{SN}}-1)^{\delta}$.

To obtain the optimal number of hops M, we relax M to be a real number, differentiate the objective function with respect to M, and set the result to zero. We have

$$\begin{aligned} & M^2 - (k_1 + qk_2)2P_{SN}^{-\delta} = 0, \\ & M^* = \left[2\left(\mu P_{PN}^{\delta}\mathbb{E}[G_{p,s}^{\delta}] + qk_0\lambda\mathbb{E}[G_{s,s}^{\delta}]\right)\pi\Gamma(1-\delta)m_{s,s}^{-\delta} \right. \\ & \quad \left. \cdot L^2\left(2^{R_{SN}}-1\right)^{\delta}P_{SN}^{-\delta}\right]^{\frac{1}{2}}. \end{aligned} \tag{12}$$

We observe that the optimal hop number M^* is proportional to $(P_{PN}/P_{SN})^{1/\alpha}$ (note that $k_0 \propto P_{PN}^{\delta}$). When the transmit power P_{SN} increases, we prefer long-hop transmissions. In addition, M^* increases linearly with respect to the end-to-end distance L. Substituting M^* into (11), the minimum average end-to-end delay can be computed as $\mathbb{E}[T]^* = M^*\sqrt{e}/(q\tilde{p})$, which is proportional to $(P_{SN}/P_{PN})^{1/\alpha}$ (note that $\tilde{p} \propto (P_{PN}/P_{SN})^{2/\alpha}$ and the non-outage probability is a constant $p^s = 1/\sqrt{e}$).

When P_{PN} increases, although the SR may suffer more interference, M^* increases (the hop-length decreases) and the SR is closer to the ST such that the received signal power at SR becomes stronger. It turns out that we have a fixed $p^s = 1/\sqrt{e}$. At the same time, as P_{PN} increases, the received signal power at PR from PT also increases since their separation d is fixed. Given the same outage constraints at PRs, more inter-system interference from STs to PRs can be tolerated and thus the maximum medium access probability of an SU (i.e., $\tilde{p}$) grows. The resulting increase in medium access probability compensates the delay from the larger hop-count and thus causes lower end-to-end delay.

On the other hand, when P_{SN} increases, the optimal end-to-end delay increases mainly due to the smaller medium access probability. We may not be able to arbitrarily reduce the delay by lowering P_{SN} since we cannot find any neighbor with arbitrarily small distance, where the lower bound becomes loose. Fig. 1 and Fig. 2 depict the optimal number of hops and the corresponding minimum average end-to-end packet transmission delay under different source-destination distances L and different cross-tier fading gains $m_{s,p} = m_{p,s}$. Linear relationships can be observed. Also, as the inter-system interference gets smaller ($m_{s,p}$ and $m_{p,s}$ decrease), short-hop transmissions are preferred and the corresponding end-to-end delay decreases.

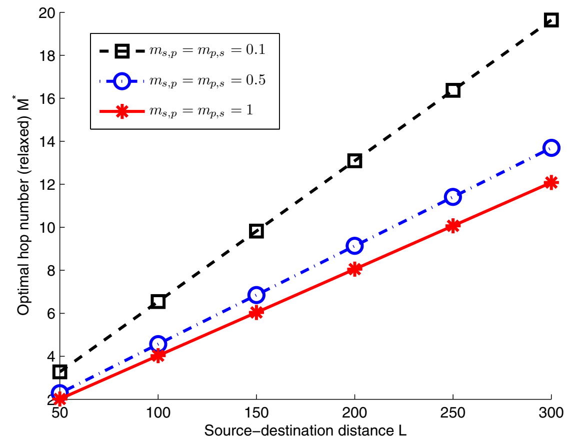

Fig. 1. Plots of the optimal number of hops M^* v.s. source-destination distance L. The system parameters are set as $\alpha = 4$, $\lambda = 10^{-3}$/m^2, $P_{SN} =$ 0.1mW, $R_{SN} = 2$ bits/s/Hz, $q = 1$, $\mu = 10^{-5}$/m^2, $P_{PN} = 0.3$mW, $R_{PN} = 2$ bits/s/Hz, $d = 15$m, and $\epsilon = 0.1$. All links are Rayleigh faded, where $m_{s,s} = m_{p,p} = 1$.

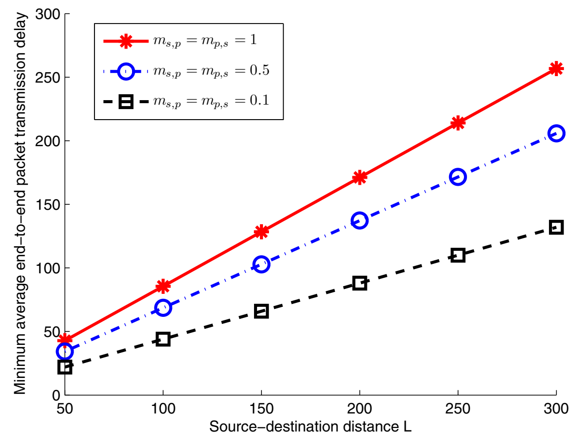

Fig. 2. Plots of the minimum average end-to-end packet transmission delay $\mathbb{E}[T]^*$ v.s. source-destination distance L. The system parameters are the same as in Fig. 1.

2) Optimal transmit power allocation: We may minimize the average end-to-end packet transmission delay by optimizing the transmit power of an SU. By differentiating the objective function in (11) with respect to P_{SN}, and setting the result to zero, the optimal power assignment of an SU

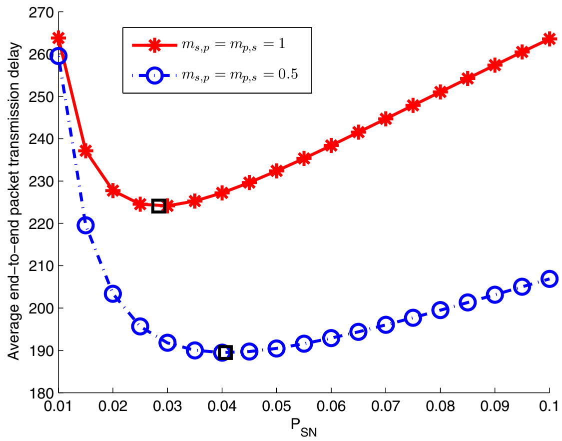

Fig. 3. Plots of the average end-to-end packet transmission delay $\mathbb{E}[T]$ v.s. transmit power of an SU P_{SN}. The system parameters are set as $\alpha = 4$, $\lambda = 10^{-3}$/m^2, $R_{SN} = 2$ bits/s/Hz, $q = 0.5$, $\mu = 10^{-5}$/m^2, $P_{PN} =$ 0.3mW, $R_{PN} = 2$ bits/s/Hz, $d = 15$m, $\epsilon = 0.1$, $L = 200$m, and $M = 6$. P_{SN} ranges from 0.01mW to 0.1mW. All links are Rayleigh faded, where $m_{s,s} = m_{p,p} = 1$.

P^*_{SN} can be computed as

$$P^*_{SN} = \left(\frac{k_1 + qk_2}{M^2}\right)^{\frac{1}{\delta}} = \left[\left(\mu P^{\delta}_{PN}\mathbb{E}[G^{\delta}_{p,s}] + qk_0\lambda\mathbb{E}[G^{\delta}_{s,s}]\right)\right.$$
$$\left.\cdot\,\pi\Gamma(1-\delta)m^{-\delta}_{s,s}L^2(2^{R_{SN}}-1)^{\delta}M^{-2}\right]^{\frac{1}{\delta}}. \quad (13)$$

We observe that P^*_{SN} is proportional to P_{PN} (note that $k_0 \propto P^{\delta}_{PN}$) and to the α power of hop-length L/M. The corresponding minimum average end-to-end delay can be derived as $q^{-1}k_0^{-1}(k_1+qk_2)e/M$. The average end-to-end delay is plotted with respect to P_{SN} in Fig. 3. We observe that the minimum is achieved at $P^*_{SN} = 0.03$ (when $m_{s,p} = m_{p,s} = 1$) and $P^*_{SN} = 0.04$ (when $m_{s,p} = m_{p,s} = 0.5$).

V. Conclusion

In this paper, we investigate the end-to-end packet transmission delay of routing in cognitive radio ad hoc networks, considering both intra-system and inter-system interference. With the outage constraints at PRs, the access probability of an SU is limited, inducing excess medium access delay. The quality of the received signal at an SU degrades due to the interference from PTs, leading to an increase in the number of retransmissions. The end-to-end delay, which consists of medium access delay, retransmission delay, and the hop count, is minimized by adjusting the number of hops of the end-to-end route, serving as a delay lower bound. The success probability of a secondary transmission is shown to be a constant $1/\sqrt{e}$ at the optimum hop-length. Our analysis illustrates the impacts of different system parameters in both primary and secondary networks on the end-to-end packet transmission delay in the secondary network, facilitating delay QoS provisioning and rate-delay trade-off.

Acknowledgments

This work is supported in part by National Science Council under contract NSC 102-2221-E-011-046-MY2. These authors would like to thank Shih-Hao Lin in Department of Computer Science and Information Engineering, National Taiwan University of Science and Technology, for the derivation of analytical model in this paper.

References

[1] Y.-C. Liang, K.-C. Chen, G. Li, and P. Mahonen, "Cognitive radio networking and communications: An overview," *IEEE Trans. Veh. Technol.*, vol. 60, no. 7, pp. 3386–3407, Sep. 2011.

[2] A. Goldsmith, S. A. Jafar, I. Marić, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proc. IEEE*, vol. 97, no. 5, pp. 894–914, May 2009.

[3] K. Chowdhury and I. Akyildiz, "CRP: A routing protocol for cognitive radio ad hoc networks," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 4, pp. 794–804, Apr. 2011.

[4] W. Ren, Q. Zhao, and A. Swami, "On the connectivity and multihop delay of ad hoc cognitive radio networks," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 4, pp. 805–818, Apr. 2011.

[5] M. Pan, C. Zhang, P. Li, and Y. Fang, "Joint routing and link scheduling for cognitive radio networks under uncertain spectrum supply," in *Proc. IEEE INFOCOM '11*, Apr. 2011, pp. 2237–2245.

[6] C.-F. Shih, W. Liao, and H.-L. Chao, "Joint routing and spectrum allocation for multi-hop cognitive radio networks with route robustness consideration," *IEEE Trans. Wireless Commun.*, vol. 10, no. 9, pp. 2940–2949, Sep. 2011.

[7] Q. Guan, F. R. Yu, S. Jiang, and G. Wei, "Prediction-based topology control and routing in cognitive radio mobile ad hoc networks," *IEEE Trans. Veh. Technol.*, vol. 59, no. 9, pp. 4443–4452, Nov. 2010.

[8] A. Abbagnale and F. Cuomo, "Leveraging the algebraic connectivity of a cognitive network for routing design," *IEEE Trans. Mobile Comput.*, 2011, to appear.

[9] A. Rabbachin, T. Quek, H. Shin, and M. Win, "Cognitive network interference," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 2, pp. 480–493, Feb. 2011.

[10] Y. Shi, Y. T. Hou, S. Kompella, and H. D. Sherali, "Maximizing capacity in multihop cognitive radio networks under the SINR model," *IEEE Trans. Mobile Comput.*, vol. 10, no. 7, pp. 954–967, Jul. 2011.

[11] C. Lee and M. Haenggi, "Interference and outage in poisson cognitive networks," *IEEE Trans. Wireless Commun.*, vol. 11, no. 4, pp. 1392–1401, Apr. 2012.

[12] F. Baccelli, B. Blaszczyszyn, and P. Muhlethaler, "On the performance of time-space opportunistic routing in multihop mobile ad hoc networks," in *Proc. IEEE WiOPT '08*, Apr. 2008, pp. 307–316.

[13] J. Andrews, S. Weber, M. Kountouris, and M. Haenggi, "Random access transport capacity," *IEEE Trans. Wireless Commun.*, vol. 9, no. 6, pp. 2101–2111, Jun. 2010.

[14] R. K. Ganti and M. Haenggi, "Bounds on the information propagation delay in interference-limited aloha networks," in *Proc. IEEE WiOpt '09*, Jun. 2009.

[15] F. Baccelli, B. Blaszczyszyn, and M.-O. Haji-Mirsadeghi, "Optimal paths on the space-time SINR random graph," *Adv. Appl. Prob.*, vol. 43, no. 1, pp. 131–150, 2011.

[16] R. Vaze, "Throughput-delay-reliability tradeoff with ARQ in wireless ad hoc networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 7, pp. 2142–2149, Jul. 2011.

[17] K. Stamatiou and M. Haenggi, "Delay characterization of multihop transmission in a poisson field of interference," *IEEE/ACM Trans. Netw.*, 2011, submitted.

[18] Y. Chen and J. Andrews, "An upper bound on multi-hop transmission capacity with dynamic routing selection," in *Proc. IEEE ISIT '10*, Jun. 2010, pp. 1718–1722.

[19] M. Haenggi, J. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, "Stochastic geometry and random graphs for the analysis and design of wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 7, pp. 1029–1046, Sep. 2009.

[20] R. Ganti and M. Haenggi, "Spatial and temporal correlation of the interference in ALOHA ad hoc networks," *IEEE Commun. Lett.*, vol. 13, no. 9, pp. 631–633, Sep. 2009.